



CDS pricing using a Copula-Monte Carlo Approach

Reyna Susana García Ruiz ¹

Facultad de Contaduría y Administración, Universidad Nacional Autónoma de México

Francisco López-Herrera

Facultad de Contaduría y Administración, Universidad Nacional Autónoma de México

Salvador Cruz-Ake

Escuela Superior de Economía, Instituto Politécnico Nacional

Recibido el 1 de noviembre de 2017; aceptado el 6 de diciembre de 2017; Disponible en Internet el 14 de diciembre de 2017.

Abstract:

This paper proposes a methodology, based on Copula, Monte Carlo, and Bootstrap methodologies, to price a CDS without using more data than the one provided by the financial statements. This means that our methodology could be suitable not only for firms listed in the exchange market but also for nonlisted firms, so the results shown on the paper could extend the possibility of pricing CDS. The propounded methodology links the default probabilities to some key variables which dependence structure is captured by a copula and recombine it for pricing the CDS. To test the validity of the proposed methodology, we used data from TV Azteca (a media Mexican Company with recent financial concerns) and we obtained a CDS spread similar to the default rate implied in its credit risk rating.

Keywords: Credit Default Swap; CDS; Copula-Monte Carlo; Default probabilities

JEL Codes: G12; G17; G32

Valuación de Credit Default Swaps CDS , una aproximación con cópulas y métodos Monte Carlo

Resumen:

El presente artículo propone una metodología de valuación basada en cópulas, simulaciones Monte Carlo y métodos de bootstrap para valorar Credit Default Swaps –CDS- empleando solamente la información financiera de la empresa. Esto significa que nuestro método es adecuado no solo para empresas cuyas acciones cotizan en una bolsa de valores, sino también para empresas privadas. Esto de tal forma que la metodología puede extenderse para valorar CDS. Nuestro modelo relaciona las probabilidades de incumplimiento de pago con algunas variables financieras clave, al emplear una copula para modelar la estructura de dependencia. Para demostrar su validez, utilizamos datos de Televisión Azteca y logramos un diferencial de CDS similar al de la tasa de incumplimiento implícita en la calificación crediticia de la compañía.

Palabras clave: Credit Default Swap; CDS; Copula-Monte Carlo, Probabilidades de incumplimiento

Códigos JEL: G12; G17; G32

¹ La autora agradece al Programa de Becas Posdoctorales en la UNAM el apoyo financiero recibido para llevar a cabo la investigación que da origen a la elaboración de este artículo.



1. Introduction

The veering economic and financial environment during recent years has profoundly affected companies of all sizes and sectors, although it has been especially pernicious for those companies with high leverage and procyclical cash flows. Indeed, the academic debate coincides with the discussion in the industry, since an optimal level of leverage is not evident when the cost of debt is variable, or when the cash flows are affected by inconstant economic conditions. These economic conditions also influence the value of the possible collaterals and the refinancing conditions of credit. For more details on the effect of market conditions on credit, see Chen (2010) or Figlewski, et al. (2012).

As a response to the consequences of the environment uncertainty in the firm's capability to repay its loans, the financial markets developed the Credit Default Swaps (CDS). This credit derivative has the purpose of transferring the credit risk to the portfolio of an economic agent who, in exchange for a premium, is willing to face the credit risk associated to the private loan taken as the underlying instrument. For more details on the CDS functioning, see Hull & White (2001) or Chen, et al. (2013).

The idea behind the CDS is that any private loan can be artificially detached into an instrument free of credit risk (a government bond on its currency) and a credit risk instrument (the payment associated with the credit risk). Interested readers in the trenching procedure may see Schönbucher (2003) or Amato (2005). When any bank makes a loan to a private company, it faces the possibility of a default on any part of the life of the instrument. If that bank enters a CDS with another economic agent (the insurer), the bank will make a series of payments that, on average, fulfill the expected value of the default. This expected value of payments will be equal to the expected cost of the credit risk instrument. Hull & White (2000) explain clearly this process.

If there is no default, the bank will pay for the credit risk to the insurer. However, if a default occurs, the bank will exchange the defaulted bond for a government bond with the same face value, and the part that issued the CDS will get the defaulted bond with all the legal rights to ask for the debt



payment. These claiming rights have the expected value of the recovery amount. Authors like Tang (2007) or Duffie (2008) provide empirical evidence about the relationship between the CDS and the traditional moral hazard component in insurance insomuch that its execution depends only on external conditions for all the related parties in the derivative.

The inclusion of a CDS may have two effects on the cost of debt. The first one is the increased monitoring in the accredited company, which reduces the possibility of bad practices that may impede the payment of the capital, interest and other payments associated with the bond. For details see Besanko (1993) or Hauswald & Marquez (2006). The other one is that the issuance of CDS may reduce the interest rate charged to compensate the adverse selection risk related to the credit issuing process and the lack of information about the firm's behavior when the economic conditions worsen.

Traditionally, only the large financial companies use the CDS as a tool for controlling the credit risk portfolios, although there are serious attempts to use this technology in the management of loans and commercial credit portfolios. Authors like Smithson (2003) or Ketkar & Ratha (2001), explains the use of the credit risk management for non-financial corporations. In their works, they use the aggregated information of similar firms to determine the default probability and the recovery rate of the examined portfolio.

In this paper, we make a proposal that uses only the information from the firm and its interaction with its environment, gathered using a copula, and we create a tailored CDS that relies only on the company's information to get the implied default probabilities through a Monte Carlo simulation. The interested readers in Monte Carlo Methods may see Mooney (1997) or Chib & Greenberg (1995). To focus the analysis on the effect of the Copula-Monte Carlo methodology, we calculated the spread rate of the CDS supposing five different recovery rates: 0%, 30%, 60%, 90%, and maintaining all the other variables equal.



Our methodology obtains its initial default probabilities using a variation on the Merton (1974) default model. We use the cash flows from operations (assuming the company does not take any decision that affects the cash flow) as the underlying of a derivative that has a strike price given by the financial need of the enterprise (interest plus amortization in the quarter) in the next quarter. We use the complement of the derivative's execution probability, to make a proxy of the implied odd for the default of the company.

After this process, we make triads with the cash flow from operations, the yield of the Global Economic Activity Indicator (IGAE) and the previously calculated default probability. With this data in hand, we proved several distributions for each marginal and, after getting the best marginals (we selected them using the Anderson & Darling (1954) test), we proved the most common copula families. We selected the copula using Anderson & Darling (1954) test for the marginal resamples and taking an arithmetic average of them to find the best copula. Even though Genest, et al. (2009) and Kojadinovic & Holmes (2009) present an excellent review on the goodness of fit and power of tests for copula, we prefer this procedure due its simplicity and accuracy for modeling.

After selecting the best copula, we calculate the value of a theoretical CDS using the resamples of the best copula with five different recovery rates (0%, 30%, 60%, 90%, and maintaining all the other variables equal) to get a result from the Monte Carlo procedure. The proposed procedure gives us the opportunity to price the CDS even if the firm is not public (it only requires data from the financial statements) and uses general economic data to relate the company's default probability to the rest of the economy and a key firm variable. In the same way, the recovery rate can be estimated using the market value of each collateral of the loan minus an estimation of the legal costs incurred to execute the warranty.



A potential use of our results is the possible reduction of the charged interest rate to the non-public firms. The possible rate reduction arises from the difference between the average rate charged by a commercial bank to the company (that includes a general spread that covers all its loan portfolio), and the sum of the risk-free rate and the CDS spread calculated not with the common data from a pool of similar firms. Instead of that pool, we used simulated data from the analyzed firm, related to the economic environment, to calculate the default probabilities, this means a tailored pricing for each firm.

The next section deals with a quick state of the art review of the influence of each parameter involved in the pricing of the CDS. Section three is devoted to the selection and use of the copula and some key aspects of the Monte Carlo procedures. Section four shows the use of the propounded methodology in order to price the CDS of a struggled Mexican firm (in this case public to use financial statements that are publicly disclosed). Finally, we conclude and sketch some future research lines in the last section of this paper.

2. Credit Default Swap Pricing

In their seminar article, Hull & White (2000) give the core methodology for pricing a CDS. There, they explain that the goal of the pricing is to obtain the spread value, ω^* , which balances the expected value of the firm's default, $1 - R^a g_t R^b$, with the present value of the payments given by the bond holder to the CDS issuer (the person that gives the protection against default). $1 - R^a g_t R^b$

Thus, the value of the CDS is:

$$\omega^* = \frac{\sum_{i=1}^N (1 - R^a g_t R^b) Q_i v_i}{\sum_{i=1}^N (u_i + e_i) Q_i - \pi u_T}, \quad (1)$$



where T is the time of maturity of the CDS, Q_i is the default probability of the bond in t_i , R_i is the recovery rate of the bond, u_t is the current value, in $t = 0$, (conditional to default) of a currency unit, associated with the payments of the CDS, e_t is the current value, in $t = t - 1$, (conditional to default) of a currency unit, associated with the payments of the CDS, v_t is the risk-free current value of a currency unit available in t , ω is the payment, at the bond periodicity, made by holder of the risky bond to the CDS issuer (short side), π is the non-default probability during the lifespan of the CDS, g_t is the accrued interest on the risky bond at time t .

We will initiate the study of the risk variables in the CDS with the default probability Q_i . In their paper, Bianchi & Fabozzi (2015) analyze several reduced methodologies to model the intensity of the default rate in a CDS. In particular, these authors study non-Gaussian stochastic processes (Levy flights with Gamma intensities, inverse Gaussians and Gamma distributions for the variance) and Sato stochastic processes to model the likelihood of default of the bond. For more details on these stochastic processes, see Tankov (2003), Applebaum (2009), Barndorff-Nielsen (1997) or Sato (2002).

The reduced models do not require too many data for its implementation, but they do not have a relation with the macroeconomic environment. To solve this problem, some authors such as Duan (2009) propose a reduced model to predict the intensity of default, based on an econometric adjustment of the probability of default. This kind of modeling adds macroeconomic realism to the simplicity of instrumentation of the classic reduced models.

The main criticism made to the reduced models is its inability to adapt to market instability, especially to the volatility clusters. In this regard, Coudert & Gex (2013) found that the CDS



premiums lead the corporate surtaxes and that this leadership increases in moments of high volatility of the financial markets. By their side, Huang & Hu (2012) used a Smooth Transition Autoregressive Models (STARS) to analyze the CDS surcharges.

Finally, we mention the probabilistic approach to the calculation of the default probability; this encompasses methodologies as varied as convolution distributions, see Duan (2010), or the use of copulas to model the occurrence of simultaneous defaults, see Liu & Morley (2013). We chose this approach because it captures all the dependence structure of the analyzed variables.

A second element to be analyzed in the study of CDS is the rate of recovery of the bond given the default. Its value is related to the current value of the collateral of the loan, the leverage of the loan and the general solvency of the issuer of the bond. In the Hull & White (2014) paper, they propose the use of stochastic differential equations with contaminations given by Poisson jumps to value the recovery rate, R_t^0

Like the default rate, some authors like Pan & Singleton (2008) model the recovery rate, R_t^0 with reduced models, while Perraudin & Hu (2006) use extreme values to model the intensity of the falls in the percentages of recovery of corporate bonds. An interesting state of the art compilation on recovery rate and its relationship with other CDS variables can be seen in Altman, et al. (2004) or Madan, et al. (2006).

Finally, it is necessary to address the possibility that the CDS issuer may not honor the CDS (because it is not a risk-free agent). In this regard, authors such as Crépey (2012), and Brigo et al. (2014), use an option pricing approach to value the possibility of the issuer failing the derivative. In a similar manner, (Arora, et al. (2012) show that the possibility of the issuer's default relies heavily



on the financial characteristics of the issuer and that the CDS is cheaper if the counterparty has a higher credit risk.

3. Copula and Bootstrap - Monte Carlo Methodology

As we explained in the introduction, our paper proposes a methodology to price a CDS even if the firm is not public, without using values of similar companies and their past defaults. Our methodology also relates the default probabilities to the company's behavior and environment; this means that we can price a CDS for any given firm (we only need the financial statements) considering its performance and some economic key variables. In this case, we use the disposable cash of TV Azteca and the yield of the GDP.

TV Azteca is a Mexican media company, based in México City. In the last months, it suffered a severe downfall on its revenues due to the competition with Televisa (the leader media company in México) and Netflix. Its financial crisis became public six months ago when the company lost 554 millions of Mexican pesos in the first quarter of 2016. Its financial distress and the fact of being a public company makes it a proper study subject for testing our procedure because we have access to its audited financial data (retrieved from Economática) and the rating agencies give approximated measurements of its default probability.

The first step in our methodology is to obtain the company's default probabilities. For doing so, we use a variation of the Merton (1974) model. This variation consists of using the operational cash flow as the underlying asset of a European call which strike price is the quarterly financial needs of the firm (interest and capital amortization payments). We also use the cash flows volatility (as a percentage of the last cash flow) and the Mexican risk-free rate to finish the pricing of the derivative



which time to expiration is always three months (when a new payment will be required, and the option is renewed).

After we did the standard calculations to price a European call option, we took the complement of the probability of exercise the option, $N(-d_2)$, as the estimated TV Azteca default probability in that quarter. With the default probabilities in hand, we formed triplets adding to each default probability the yield of the GDP and the value of the firm's disposable cash (the method relies on the existence of the triplet, not in the variables). As a future research line; we may make some research on alternative methods to get the default probabilities as those showed in the works of Westgaard & Van der Wijst (2001), Rösch (2005) or Nickell, et al. (2007).

The future research also may be extended to the use of other variables in the n-tuples or the use of other distribution functions for the default probability. Authors as Bharath & Shumway (2008) or Lopez (2004) give some insights in this research lines.

Once we formed the triplets, we fitted a distribution function to each one of them. We selected the marginal distribution using an Anderson-Darling (AD) test; for more details on the AD test, the interested reader may see Anderson & Darling (1954), Razali & Wah (2011) or Scholz & Stephens (1987). Once we calculated the default probabilities, we needed to relate them. For doing this, we use a copula approximation that catches and reproduces the interdependence relation between the variables.

The use of the copula approach is not new in Risk Management. The works of Daul, et al. (2003) and Frey, et al. (2001) are excellent examples of the first steps in its use. More recent examples are those published by Oh & Patton (2016) or Harb & Louhichi (2016).

Broadly speaking, a copula is a multivariate probability density function whose domain is a unit hypercube of dimension n , equals to the number of marginals into the function. Although the variety of copula functions is wide, all of them fulfill the properties stated in the seminal work, that of Sklar (1959) or in more recent times Rüschendorf (2013). As stated in the introduction, the copula function is selected using the average goodness of fit of the copula-generated n -tuples compared to the original data.

In this paper, we use a normal copula with a Generalized Extreme Value (GEV) distribution fitted to the IGAE yield, a normal distribution fitted to the firm's default probability, and a Weibull distribution fitted for the disposable income values, this is

$$C_{\text{normal}}(u_1, u_2, u_3) = \frac{1}{\sqrt{\det R}} e^{\left(-\frac{1}{2} \begin{pmatrix} \Phi(u_1) \\ \Phi(u_2) \\ \Phi(u_3) \end{pmatrix}^T (R^{-1} - I) \begin{pmatrix} \Phi(u_1) \\ \Phi(u_2) \\ \Phi(u_3) \end{pmatrix} \right)}, \quad (2)$$

where R is the correlation matrix, and I is the identity matrix,

$$u_1 = F(x)^{-1} = e^{\left\{ -\left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\}}, \quad u_2 = F(x)^{-1} = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2}, \quad \text{and} \quad u_3 = F(x)^{-1} = 1 - e^{-\left(\frac{x}{\lambda} \right)^k}, \quad \forall x \geq 0. \quad (3)$$

To estimate the copula, we fitted the corresponding distribution probabilities using the “ismev” Heffernan & Stephenson (2016) and “MASS” Venables & Ripley (2002) packages. Then, we obtained the probabilities of each realization, u_i , with the “fExtremes” Wuertz & et. al. (2013), “stats” R Core Team (2015) packages to feed the copula.

The “copula” Hofert, et al. (2015) package fit the desired normal copula using a two stages maximum likelihood procedure; the Kojadinovic & Yan (2010) paper explains in detail this



procedure. After fitting the copula, we make 10,000 simulations and then export them to a csv archive using the “utils” R Core Team (2015) package.

After generating enough artificial *triplets* (10,000) for the Monte Carlo simulation, we price the CDS following the methodology proposed in Hull & White (2000). Since we are trying to pricing the CDS without using external data, we do not use an average recovery rate taken from external data. Instead, we price it varying the recovery rate R% parameter (0%, 30%, 60%, and 90%) and taking the average CDS spread values for each scenario.

The possibility of relating some external values to the default probability of the firm gives the opportunity of making a straightforward stress test to the CDS. In fact, the researcher may choose the desired economic conditions and price the CDS, or simply use an external model to make some guesses on the future economic circumstances and then obtain scenarios for the value of the CDS.

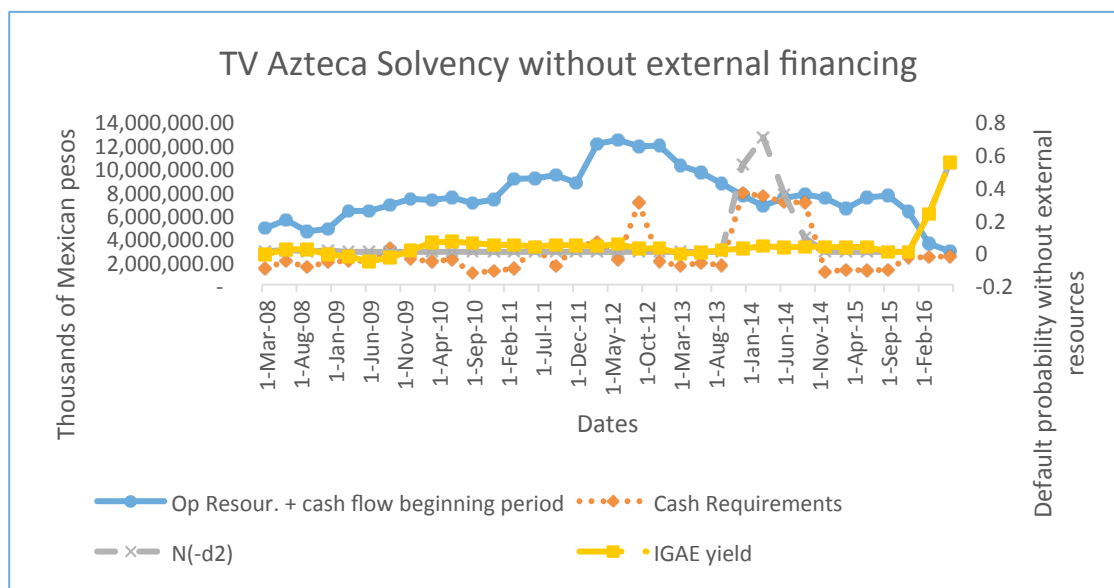
In this paper, we will use all the values obtained from the copula simulation to price the CDS using the Bootstrap Efron (1979) algorithm, see García-Sánchez & Cruz-Ake (2015), that preserves the actual interest rates for corporate and government loans. The idea behind using the Bootstrap methodology is to recreate “new possible outcomes” based on real values. Essentially, the Bootstrap methodology consists on resampling the data and use the “plug-in estimator” assumption. For more details and advances in the Bootstrap methodology, the interested reader may see Chernick (2011) or Davison & Hinkley (1997).

By combining both procedures, the paper gives to the user the chance of making a stress test focused on certain scenarios while maintaining the true relations between the rest of the variables. In the next section, we will show the current situation of TV Azteca, its calculated default probabilities, the results of the copula simulations, and the CDS pricing.

4. CDS Pricing for TV Azteca

As mentioned previously, TV Azteca S.A.B. de C.V. is a Mexican media company, listed in the Mexican Stock Exchange under the ticker AZTECACPO.MX. For the analysis, we retrieved its quarterly financial information from March 2000 to September 2016. As we stated previously, we calculated the default probabilities using a variation of the Merton (1974) methodology. We show these probabilities and their relation to the relevant variables in Figure 1.

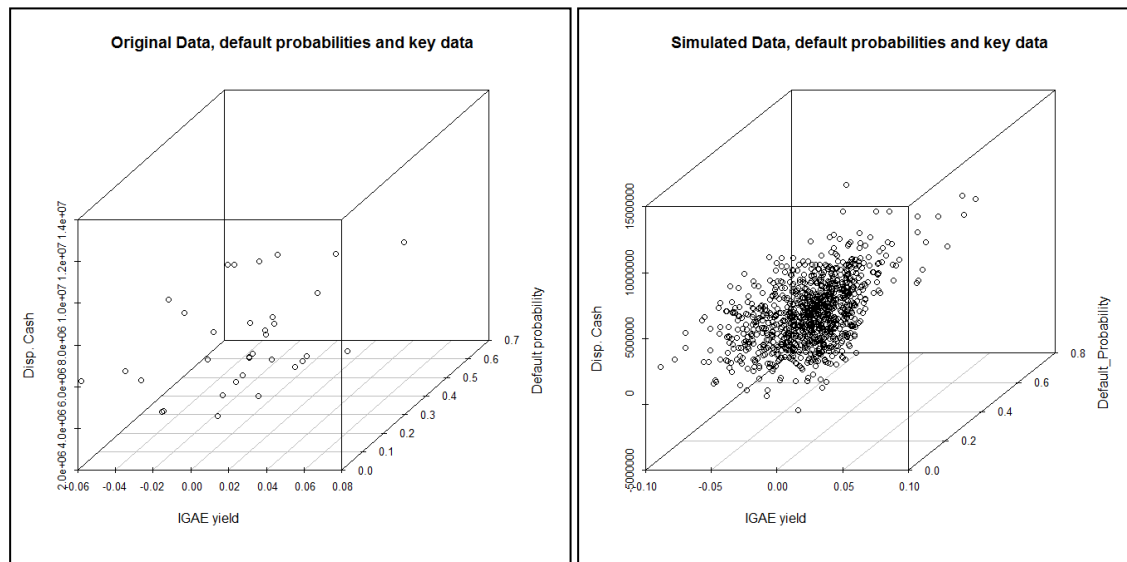
Figure 1 TV Azteca solvency and key variables. Own elaboration with data from Economatica e INEGI



Fuente. Elaboración Propia.

After these calculations, we made the marginal and copula fitting. We show the original data and the reproduced data in figure 2, to demonstrate the accuracy of the calculations. As the reader may realize, the main difference between the simulations and the original cube is the amount of data (simulations look more populated), but the limits of the variables and the shape of the data cloud are almost the same.

Figure 2 Original and simulated data from the Copula. Own elaboration using R.



Fuente. Elaboración Propia.

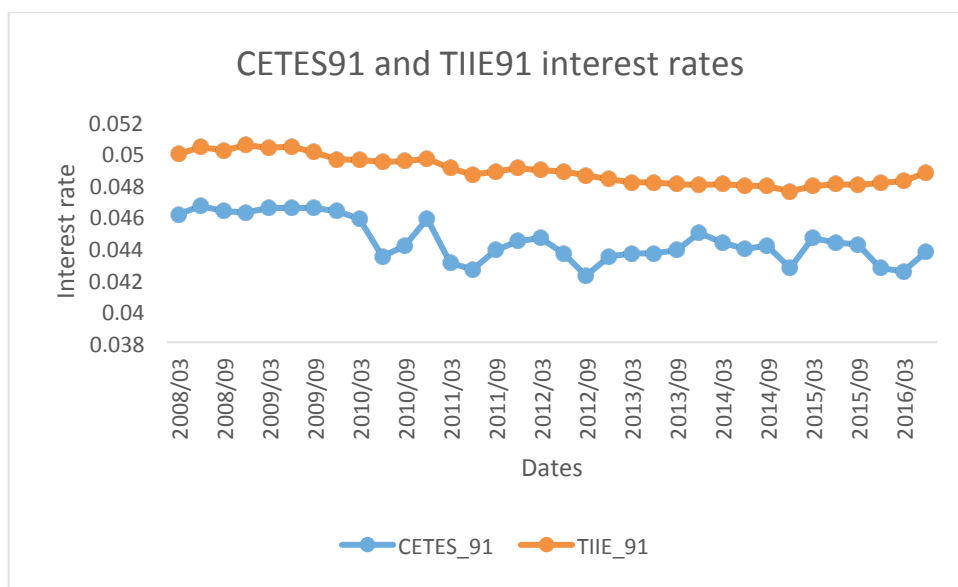
We transferred all the simulated triplets into a VBA algorithm García-Sánchez & Cruz-Ake (2015) that calculates the value of the CDS using a Bootstrap procedure Efron (1979). The algorithm uses actual values of the Mexican CETE and the TIIE as the risk-free and balance interbank interest rates respectively (to maintain the relations between both rates). We show them in Figure 3.

The theoretical bond covered by the CDS is a semi-annual bond with a five-year time to maturity and a face value of 100 Mexican pesos. We took the characteristics of the bond from a traditional medium to long term private debt issued by a large Mexican firm. We show in Table 1 the results of the pricing for the general and stressed cases with various recovery rates (0%, 30%, 60%, and 90%).

The reader may see in Figure 1 that the default probabilities pikes when the operational resources and the cash requirements also pike. Finding a default probability not so closely related to the disposable cash is a surprising fact. Although the default probability decreases when the disposable cash is above the 80% percentile, it decreases slowly. Table 1 shows this effect, suggesting that some other hidden factors conduct the default probability.

Despite the problems of closely relating the disposable income to the default probability, we may point out that the CDS spread of the general sampling (4.55%), given a zero-recovery rate, is very near to the Cantor, et al. (2007) default rate implied by the Fitch B+ current rating for TVAzteca (2.11% - 8.82%).

Figure 3 Risk-free and corporate base rate in México. Own elaboration with data from México's Central Bank (BANXICO).



Fuente. Elaboración Propia.

Table 1 CDS spread for TV Azteca using our methodology. Own elaboration with data from Economica and BANXICO

CDS spreads for different recovery rates under different disposable income scenarios				
Recovery Rate	0%	30%	60%	90%
General CDS spread	0.04552985	0.03193185	0.0183634	0.00458595
80% percentile CDS spread	0.03086224	0.02204361	0.01262929	0.00309957

Fuente. Elaboración Propia.



5. Conclusiones

This paper proposes a methodology, based on Copula and Monte Carlo methodologies, to price a CDS without using more data than the one provided by the financial statements. So, the methodology shown in the paper could be also suitable for nonpublic traded firms, expetedly without concerns related to its size or industry. The proposed methodology links the default probabilities to some key variables that can be selected by the user, and which dependence structure is captured by a copula.

The propounded methodology overcomes the need of using data from other similar companies (same economic sector, size, and perceived credit risk) that may create a bias in the analysis by creating as many resamples as needed from the fitted copula, even if the study is focused on a small subsample of the original data. Our methodology also allows the researcher to focus its work on a certain part of the data to perform a stress test by using one of those key variables as a control variable. We showed that the outcomes of the copula resample were very similar to the original data in scale and shape. This accuracy in modeling the default rate and its relation to the key variables allows us to price the CDS spread correctly.

We proved our methodology with a publicly traded, Mexican company that presented financial problems (TV Azteca). This company has a current B+ credit score from Fitch ratings, which is in line with our calculated 4.55% spread for the zero-recovery rate in the extensive sampling.

Although the proposed methodology seems to work properly, it can be extended in several directions. The first extension is to prove higher dimensional copulas (for example vine copulas) or time-dependent copulas. Another possible extension is using a different set of key variables that may be more correlated to the default probability. To include the risk-free rate in the copula is another possibility for extending the paper.



The main potential contribution of this article is the better credit risk measurement for a nonpublic traded company; this better measurement may create an arbitrage opportunity for the banks or the market borrowers that lowers the cost of the credit for these enterprises. We claim that the same methodological device shown in this paper could also be applied to the risk measurement of the CDS spreads for sovereign bonds using only data from the analyzed country; needless to say, this remains as a possible line for future research.

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